Topic Study Group 16

Reasoning, Argumentation and Proof in Mathematics Education

Viviane Durand-Guerrier[1],Samuele Antonini[2], Kotaro Komatsu[3],Nadia Azrou[4] and Chao Zhou[5]

1. Aims of the TSG

There is international recognition (see Stylianides and Harel, 2018) of the importance of reasoning and proof in students' learning of mathematics at all levels of education (elementary, secondary, university) and in all tracks (general, vocational). Indeed, reasoning, argumentation and proof are at the very heart of mathematical activity, playing a crucial role in learning processes. There is also international research-based evidence showing that many students face difficulties with reasoning about mathematical ideas and constructing or understanding mathematical arguments. This is particularly the case when these arguments meet the standard of proof; in addition, teachers often lack adequate resources for helping their students to develop skills in reasoning, argumentation, and proof. Although the existing body of research offers important insights into this area, there are still many open questions for which theoretical and empirically based responses are needed.

We have welcomed submissions of theoretical or empirical research reports on any topic related to reasoning, argumentation, and proof in mathematics education, including interaction between mathematics and other disciplines (e.g., Computer Sciences, Physics, Economy etc.). The reports could cover any level of education: elementary, secondary and university (including pre-service teacher education, or in-service teacher professional development).

2. Submissions

We received 45 submissions (38 papers and 7 posters) from 17 countries: Algeria: 1; Brazil: 3; Cameroun: 1; Canada: 1; Chile: 1; China (including Hong Kong SAR and Chinese Taipei): 7; Colombia: 1; Germany: 4; Italy: 2; Japan: 4; Norway: 2; Peru: 1;

[1] University of Montpellier, France. E-mail: viviane.durand-guerrier@umontpellier.fr
[2] University of Florence, Italy. E-mail: samuele.antonini@unifi.it
[3] University of Tsukuba, Japan. E-mail: komatsu.kotaro.ft@u.tsukuba.ac.jp
[4] University Yahia Fares of Medea, Algeria. E-mail: nadiazrou@gmail.com
[5] Soochow University, China. E-mail: zhouchao@suda.edu.cn

South Korea: 2; Tunisia: 1; Turkey: 2; United Kingdom: 1; United States of America: 11. Among the 38 submitted papers, 12 were accepted as long oral presentations (Algeria: 1; Cameroon: 1; Canada: 1; China: 1; Germany: 1; Italy: 1; Japan: 1; Norway: 1; United Kingdom: 1; United States: 2; Tunisia: 1) and 26 were accepted as short oral presentations, and the seven submitted posters were accepted. During the conference, in July 2021, there were only 10 long oral presentations[1–10], and 18 short oral presentations[11–28]; during the poster sessions, 2 posters[29,30] have been presented (see Tab. 1 on the next page).

3. Sessions

Considering the high number of submissions, the ICMI organizing committee granted our TSG one more time slot for presentations. Each long oral presentation lasted 10 min and was followed by 5 min discussion; the short oral presentations were 5 minutes each followed by a collective discussion. Due to the pandemic, the sessions were in hybrid form with a small number of participants and presenters in Shanghai, the majority attending online.

In the first session on July 13th, 2021, after the introduction of the team and of the agenda of the TSG, there were two long oral presentations by Azrou[1] and Chellougui[2], and five short oral presentations by Bae[11], Na and Knuth[12], Meyer et al.[13], Solar et al.[14], and Lin[15]. A 20-min discussion on the short oral presentations followed. The session was chaired by Kotaro Komatsu (online).

In the second session on July 14th, 2021, there were 2 long oral presentations by Jablonski and Ludwig[3] and Soldano[4] and 6 short oral presentations by Shibata and Misono[16], Kempen[17], Lee[18], Dallas[19], Damrau[20], and Murata[21], followed by a 20-min discussion on the short oral presentations together with a 10-min discussion on general issues from the two first sessions in order to prepare the final collective discussion. This session was chaired by Nadia Azrou (online).

In the third session on July 16th, 2021, there were three long oral presentations by Yan and Hanna[5], Zhuang and Conner[6], Buchbinder and Crone[7] (US), and four short oral presentations by Hao and Lin[22], Huitzilopochtli er al.[23], Wong[24], and Mazzi[25], followed by a 20-min collective discussion on the short oral presentations. This session was chaired by Samuele Antonini (online).

In the fourth session on July 17th, 2021, there were three long oral presentations by Makino[8], Gustavsen et al.[9], Stylianides and Stylianides[10], and three short oral presentations by Dong and Liu[26], Zheng and Cheng[27], and Zhang and Wu[28], followed by a 20-min collective discussion on the short oral presentations, and a 45-min collective discussion on future research agenda and possible collaborations. This session was chaired by Viviane Durand-Guerrier (online) and Chao Zhou (from Shanghai).

Obayashi[29] and Barut[30] presented their posters during the related session.

Tab. 1. List of papers and posters presented

Paper and author(s)	
[1]	Writing a proof text at the university level: the role of knowing what a proof is. ***Nadia Azrou*** (Algeria).
[2]	Formalisation of proof: a tool for researcher. ***Faïza Chellougui*** (Tunisia).
[3]	Changes in the argumentation characteristics of mathematically gifted students — a longitudinal study. ***Simone Jablonski*** *and Matthias Ludwig* (Germany).
[4]	An inquiring-game for discovering and proving a geometric theorem. ***Carlotta Soldano*** (Italy).
[5]	Computer-assisted proving in the classroom. ***Xiaoheng (Kitty) Yan*** *and Gila Hanna* (Canada).
[6]	An application of habermas' theory of validity claims for classroom-based argumentation. ***Yuling Zhuang*** *and Anne-Marie Conner* (USA).
[7]	Characterizing mathematics teachers' proof-specific knowledge, dispositions and classroom practices. ***Orly Buchbinder*** *and Sharon Mc Crone* (USA).
[8]	Cognitive characteristics generating incomplete proof: analyzing the solving process of a geometrical problem by Japanese ninth graders. ***Tomohiko Makino*** (Japan).
[9]	Caught in-between tensions in teaching proof and proving. *Sikunder Ali,* ***Trond Stoelen Gustavsen****, Sigurd Johannes Hals, Andrea Hofmann, and Silje Trai* (Norway).
[10]	Posing new researchable questions as a dynamic process: the case of research on students's justification schermes. ***Andreas Stylianides*** *and Gabriel Stylianides* (UK).
[11]	Student interpretation of diagram in hyperbolic geometry: changes in the ontology of Geometric models. ***Younggon Bae*** (South Korea).
[12]	A comparative study of example uses in the proving-related activities of Korean and American students. ***GwiSoo Na*** (South Korea) *and Eric Knuth* (USA).
[13]	When is an argument an argument? Area-specific aspects of argum*ents reception.* ***Michael Meyer****, Christoph Koerner, and Julia Rey* (Germany).
[14]	Articulation of argumentation and mathematical modelling in the math classroom. ***Horacio Christian Solar*** (Chile)*, Manuel Goizueta* (Italy)*, Maria Aravena-Diaz* (Chile)*, and Andres Ivan Ortiz Jimenez* (Chile).
[15]	Fostering third graders fraction conceptions through argumentation and technology. ***Ho-Chieh Lin*** (USA).
[16]	Is there any difference in students' descriptions due to direction differences in a deductive reasoning task? ***Yoshiki Shibata*** *and* ***Tadashi Misono*** (Japan).
[17]	Investigating the differences between generic proofs and purely empirical verifications. ***Leander Kempen*** (Germany).
[18]	Proof and reasoning in high-stakes testing systems: the senior secondary mathematics curricula in Hong Kong and international baccalaureate diploma programme. ***Chun-Yeung Lee*** (UK).
[19]	Mathematics classroom argumentation: an international perspective. ***Markos Dallas*** (Norway).
[20]	Understanding the generality of mathematical statements and the role proofs play. ***Milena Damrau*** (Germany).
[21]	The function of definition in Japanese textbooks. ***Shogo Murata*** (Japan).
[22]	A comparative study of geometric proof opportunities in Chinese Taipei and Mainland middle school textbooks. ***Lei Hao*** *and P-Jen Lin* (Chinese Taipei).
[23]	Using writing and discussions to support mathematical arguments in early algebra. ***Salvador Huitzilopochtli****, Daniel Lopez-Adame and Judit Moschkovich* (USA).
[24]	Justifications in exposition in algebra in school mathematics textbooks in Hong Kong. ***Kwong Cheong Wong*** (Hong Kong SAR, China).
[25]	Different types of reasoning in geometry in Brazilian high school mathematical textbooks. ***Lucas Carato Mazzi*** (Brazil).
[26]	Analysis of analogical reasoning exercises in primary school mathematics textbook: taking geometry field as an example. ***Yaoyao Dong*** *and Jian Liu* (China).
[27]	Regional and gender differences in Chinese 8th grade students' mathematical reasoning competency. ***Xin Zheng*** *and Jing Cheng* (China).
[28]	A study of the teaching process of mathematical concept argumentation based on tap - taking function concept teaching between expert and novice teacher in China as a case. ***Yi Zhang*** *and Xiaopeng Wu* (China).
[29]	The transient stages of inductive and deductive reasoning. ***Masanori Obayashi*** (Japan).
[30]	The Last Decade of Proportional Reasoning: A Systematic Review. ***Betül Barut*** (Turkey).

4. Collective Discussions and Future Research Agenda

We have tried to keep, as possible as we could, the collective discussion along the sessions despite the tiny time due to the high number of presentations. In the last session, a 45-min slot was dedicated to discussion on future research agenda. The main issues that emerged from the presentation and discussion in perspective of future research agenda were the followings:

- The need for going on exploring epistemological and philosophical references in mathematics education, considering the increasing role of digital technologies in mathematical activity, including the role of computer assisted provers.
- The role of logic in proof and proving has long been a controversial issue in mathematics education; the body of research has been developing during the last decade and need to be still developed, considering both the use of logic in the teaching and learning of proof, and the role of logic for analyzing students' proving activities.
- The value of symbolism and of formalization in proof and proving and their interplay with heuristic aspects.
- The scope and the role of generic proof in mathematics education is still under research: in which respect such generic proofs could be recognized as *genuine mathematical proofs* by teachers is one among the open questions.
- Examining argumentation and proof in textbooks is not an easy task, because the importance of students' activities and exchanges is crucial in argumentation and proving. Nevertheless, such analysis could inform on what is likely to happen or not in classroom.
- Another important issue concerns the teacher's knowledge related to argumentation and proof, this being related with teachers' own practices, and then with their training in proof and proving in their studies; this last point being related to the double Klein transition, from secondary school to university and then back from university to secondary school.

References

A. J. Stylianides and G. Harel (2018). *Advances in Mathematics Education Research on Proof and Proving. An International Perspective*. Springer, ICME-13 Monographs.